A symmetry for the cosmological constant

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ABSTRACT: We study a symmetry, schematically Energy $\rightarrow$ — Energy, which suppresses matter contributions to the cosmological constant. The requisite negative energy fluctuations are identified with a "ghost" copy of the Standard Model. Gravity explicitly, but weakly, violates the symmetry, and naturalness requires General Relativity to break down at short distances with testable consequences. If this breakdown is accompanied by gravitational Lorentz-violation, the decay of flat spacetime by ghost production is acceptably slow. We show that inflation works in our scenario and can lead to the initial conditions required for standard Big Bang cosmology.

Contents

1. Introduction

It is sometimes hoped that the mysteries of short-distance gravity will help solve the notorious Cosmological Constant Problem [1]. However, it is difficult to see where the opportunity lies. The Feynman diagrams for matter renormalization of the cosmological constant involve only the couplings of long wavelength gravitational fields to the quantum Standard Model (SM), the domain in which General Relativity appears to work perfectly well. (See for example the discussion in ref. [2].) Another hope has been to find a symmetry under which a small cosmological constant is natural. But the most obvious candidates, supersymmetry and conformal invariance, appear too badly broken in Nature to serve this purpose. In this paper, we study a scenario in which both hopes may be realized. We employ a discrete symmetry to suppress the cosmological constant, but one which leads to instabilities of flat spacetime via gravitational processes. Adequate suppression of these processes requires a drastic breakdown of General Relativity at shorter distances.

The discrete symmetry, described in the next section, leads to an effective Lagrangian essentially the same as that proposed in ref. [3],

$$\mathcal{L} = \sqrt{-g}\{M_{Pl}^2 R - \rho_0 + \mathcal{L}_{\text{matt}}(\psi, D_\mu) - \mathcal{L}_{\text{matt}}(\hat{\psi}, D_\mu) + \cdots\}, \tag{1.1}$$

where $g_{\mu\nu}$ is the metric, ψ denotes a set of "visible sector" fields, including the SM, and $\hat{\psi}$ denotes an identical copy of the visible fields.[1] The matter Lagrangian *function*, $\mathcal{L}_{\text{matt}}$, is the same in both terms in which it appears, but with different arguments. We refer to the $\hat{\psi}$ as the "ghost sector" because of the "wrong" sign in front of its Lagrangian, including kinetic terms. Note that the ψ and the $\hat{\psi}$ include separate sets of gauge fields and separate sets of charged matter. We will discuss the ellipsis in the next section. The central point of eq. (1.1) is that the visible and ghost sectors have equal and opposite vacuum energies, canceling in their contribution to the cosmological constant, leaving only the bare (and possibly small) constant, ρ_0. This idea has an obvious shortcoming, namely instabilities originating from the ghost sector. In this paper, we nevertheless take the idea seriously by identifying the underlying symmetry, analyzing quantum effects and deducing the features required to make it a controlled and realistic scenario.

Because of negative energy excitations in the ghost sector, the Minkowski "vacuum" cannot be the ground state. However, as long as positive and negative energy fluctuations are completely decoupled, the Minkowski vacuum is stable. But with any coupling between the two, the vacuum can spontaneously decay into combinations of positive and negative energy states. Since kinematics alone do not prevent arbitrarily large mass particles being produced in such processes, even effective field theory breaks down, becoming useful only to the extent that the coupling between positive and negative energy fluctuations is very weak. See ref. [6, 7] for an earlier discussion of vacuum decays in the presence of ghosts.

In the presence of gravity all excitations of matter are necessarily coupled. The need to prevent excessively rapid decay of the vacuum requires the graviton momentum, p^{μ}, to be cut off in the ultraviolet. It is not enough to restrict the off-shell Lorentz invariant p^2; in fact, the components of p^{μ} must be cut off (in the rest frame of the universe), thereby necessitating gravitational Lorentz violation [7]. Violation of Lorentz invariance in a gravitational theory is particularly challenging because Lorentz invariance is part of the gauge symmetry of General Relativity. It is well known that breaking a gauge symmetry explicitly converts "pure gauge" degrees of freedom into physical degrees of freedom in the infrared. Any such new infrared physics must somehow not interfere with all current tests of General Relativity.

Nevertheless, explicit effective field theories have been constructed in which gravitational Lorentz violation is realized by a generalization of the Higgs mechanism [8, 9]. The leading effects on the infrared in these theories is to modify the dispersion relation of the graviton and to add additional light degrees of freedom. For a broad range of parameters, these modifications easily avoid current bounds on deviations from General Relativity. Subleading effects in the infrared, namely higher dimensional operators, can have a growing impact in the ultraviolet on the graviton dispersion relation. It is at least possible to imagine the existence of ultraviolet completions of such effective theories in which large graviton momentum transfers are highly suppressed. In this paper, we will make this admittedly strong assumption.

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[1] A later proposal in a similar vein [4, 5] introduced a symmetry under which fields are exchanged between two independent spacetimes. However, the extension of the theory to the quantum regime appears to be quite problematic.

The initial conditions for successful Big Bang cosmology require an essentially empty ghost sector, which can arise from an early inflationary phase [3].

Quantum gravity gives corrections to the perfect cancellation of vacuum energies of visible matter and ghosts. Adequate suppression of these contributions to make the observed cosmological constant natural very likely requires a breakdown in the gravitational force close to present experimental limits from sub-millimeter tests of Newton's Law [10 – 13]. Future tests should be able to probe this breakdown. The possible connection between the resolution of the cosmological constant problem and the sub-millimeter scale was first made in ref. [14]. Refs [15, 2, 16] proposed that this resolution is realized by de-localizing the gravitational interaction with matter on this scale, in a manner consistent with the equivalence principle. The technical connection between this proposal and the present paper will be discussed in future work [17].

This paper is organized as follows. In section 2, we motivate our effective Lagrangian from the viewpoint of a visible/ghost matter discrete symmetry called "energy-parity", explicitly (but weakly) broken by gravitational dynamics. In section 3, we study quantum dynamics of matter in a fixed gravitational background and show that there is no instability from negative energies at this level, and that the matter contributions to the cosmological constant naturally cancel due to energy-parity. In section 4, we estimate the quantum gravitational corrections to the cosmological constant and (assuming naturalness) use them to bound the cutoff scale on General Relativity. In section 5, we show that as long as gravitational Lorentz violation occurs at not much shorter distances than the cutoff of General Relativity, the instability of flat spacetime due to the negative energy fluctuations is consistent with observation. In section 6, we discuss the classical laws of gravity in the presence of negative energy fluctuations. In section 7, we discuss our symmetry mechanism for controlling the cosmological constant when there are metastable vacua in the matter sector. In section 8, we show how inflation can naturally lead to the cosmological initial conditions needed for our scenario. Finally, section 9 discusses our results.

2. Energy-parity

In order to motivate eq. (1.1) from a symmetry point of view, we begin by neglecting gravity and formally consider a $\mathbb{Z}_2$ "energy-parity" symmetry operation P, with $P^2 = 1$, acting on the matter Hilbert space. However, instead of commuting with the Hamiltonian, H, like a standard symmetry operator, energy-parity satisfies

$$\{H, P\} \equiv HP + PH = 0. \tag{2.1}$$

Thus, an energy eigenstate,

$$H|E\rangle = E|E\rangle, \tag{2.2}$$

is transformed into one with the *opposite* energy,

$$HP|E\rangle = -EP|E\rangle, \tag{2.3}$$

rather than a state degenerate with $|E\rangle$, as is the case for standard symmetries. We will implement this parity so a Poincaré-invariant state exists which is also energy-parity invariant, namely $P|0\rangle = |0\rangle$. From this follows

$$\langle 0|\{H, P\}|0\rangle = 2\langle 0|H|0\rangle = 0. \tag{2.4}$$

This corresponds to a vanishing cosmological constant contribution when gravity is turned back on.

Our fields transform under energy-parity in the following way:

$$\begin{aligned} g_{\mu\nu}(x) &\to g_{\mu\nu}(x) \\ \psi(x) &\leftrightarrow \hat{\psi}(x). \end{aligned} \tag{2.5}$$

Naively, it would appear that the pure gravity sector respects energy parity in eq. (1.1), while the matter Lagrangian maximally violates it. However, the opposite is true. To see this, ignore gravity and note that eq. (2.5) must be accompanied by

$$H \to -H, \tag{2.6}$$

in order to satisfy eq. (2.1). Relating the Hamiltonian to the Lagrangian,

$$\begin{aligned} L &= \int d^3\vec{x}(\Pi\dot{\psi} + \hat{\Pi}\dot{\hat{\psi}}) - H \\ &= \int d^3\vec{x}(\Pi\frac{\delta H}{\delta\Pi} + \hat{\Pi}\frac{\delta H}{\delta\hat{\Pi}}) - H, \end{aligned} \tag{2.7}$$

we see that the Lagrangian and action should be *odd* under eq. (2.5), in order to respect energy-parity. Our matter action respects energy-parity, and the gravity action maximally and explicitly violates it.

Energy-parity alone does not preclude direct matter couplings between ψ and $\hat{\psi}$, considered part of the ellipsis of eq. (1.1). Such visible-ghost couplings must be present at some level since they receive contributions induced by quantum gravity loops. These couplings, if present, would contribute to the decay of the vacuum. However, if we assume these couplings have their minimal natural strength, they do not dominate any of our vacuum decay estimates in section 5 and we will thus ignore them. The remaining terms in the ellipsis of eq. (1.1) are purely gravitational higher derivative terms. Again, their effects do not dominate any estimates in this paper.

3. Fixed gravitational background

Here, we study matter dynamics in a *fixed* soft (low-curvature) gravitational background. At the purely classical level, the negative sign in front of the ghost sector Lagrangian poses no problem, because classically the sign of the Lagrangian is physically irrelevant and the two sectors are completely decoupled without dynamical gravity. Assuming all neutrinos

have mass (entirely for simplicity of exposition), the effective theory in the far infrared is

$$\mathcal{L}_{\text{eff}} = \sqrt{-g}\left\{-\frac{1}{4}F_{\mu\nu}^2 - \rho_{\text{vis}} + \frac{1}{4}\hat{F}_{\mu\nu}^2 + \rho_{\text{vis}}\right\}$$

$$= \sqrt{-g}\left\{-\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{4}\hat{F}_{\mu\nu}^2\right\}. \tag{3.1}$$

The matter vacuum energy contributions cancel between the visible and ghost sectors because of energy-parity.

Now consider the same situation in quantum field theory. As a warm-up we simplify to the case where ψ denotes a single real scalar field rather than the entire visible sector. Similarly, $\hat{\psi}$ denotes a single ghost scalar. Further, we simplify the gravitational background to be exactly Minkowski space, $g_{\mu\nu} = \eta_{\mu\nu}$. The leading matter Lagrangian can then be written,

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\psi)^2 - \frac{1}{2}m^2\psi^2 - \lambda\psi^4 - \frac{1}{2}(\partial_\mu\hat{\psi})^2 + \frac{1}{2}m^2\hat{\psi}^2 + \lambda\hat{\psi}^4, \tag{3.2}$$

and the quantized Hamiltonian density is given by

$$\mathcal{H} = \frac{1}{2}\Pi^2 + \frac{1}{2}(\triangledown\psi)^2 + \frac{1}{2}m^2\psi^2 + \lambda\psi^4 - \frac{1}{2}\hat{\Pi}^2 - \frac{1}{2}(\triangledown\hat{\psi})^2 - \frac{1}{2}m^2\hat{\psi}^2 - \lambda\hat{\psi}^4. \tag{3.3}$$

This is the sum of two decoupled Hamiltonians. We can quantize both sub-sectors, with positive energies propagating forwards in time for ψ and negative energies propagating forwards in time for $\hat{\psi}$. Even corrected by interactions, the zero-point energies of ψ and $\hat{\psi}$ cancel, leaving zero net vacuum energy. Again, because the two sectors are completely decoupled, no pathology exists in the negative energy sector. From the viewpoint of that sector, we have merely renamed Energy ($\mathcal{H}$) by $-$ Energy ($-\mathcal{H}$).

Finally, let us consider our real case of interest, general quantum visible matter in a soft gravitational background. Here, it is easier to use path integral methods. Because the dynamics in the two matter sectors are decoupled in the absence of gravitational dynamics, the partition functional factorizes,

$$\mathcal{Z} = \left(\int \mathcal{D}\psi\; e^{i\int \sqrt{-g}\,\mathcal{L}_{\text{matt}}(\psi,D_\mu)}\right)\left(\int \mathcal{D}\hat{\psi}\; e^{-i\int \sqrt{-g}\,\mathcal{L}_{\text{matt}}(\hat{\psi},D_\mu)}\right), \tag{3.4}$$

where now ψ and $\hat{\psi}$ are a generic set of interacting fields. The opposite sign of the ghost Lagrangian now appears as the the replacement $i \to -i$ in the path integral phase factor. Since we want to propagate positive energies forward in time in the visible sector, we choose the usual "$+i\epsilon$" prescription, while propagating negative energies forward in time in the ghost sector requires a "$-i\epsilon$" prescription. All other factors of "i" in effective quantum field theory can be eliminated from the Feynman rules in position space by working exclusively in terms of real fields and couplings (taking real and imaginary components of any complex fields)[2]. Thus, the two matter sectors have identical position space Feynman rules except

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[2]There is a subtlety in the case of fermions. Here we can work in terms of real Grassman fields, χ. However, because of their anti-commuting nature, bilinears made from them must be multiplied by "i" to be Hermitian, $(i\chi_1\chi_2)^\dagger = i\chi_1\chi_2$. Thus, terms in the Lagrangian $\sim \chi^{4n+2}$ will have a factor of "i" that

for the replacement $i \to -i$ everywhere. If we integrate out some high energy physics (symmetrically) from each of the matter sub-sectors, we must get a local effective theory of the form,

$$\mathcal{Z}_{\text{eff}} = \left(\int \mathcal{D}\psi_{\text{IR}} \ e^{i \int \sqrt{-g}\mathcal{L}_{\text{eff}}(\psi_{\text{IR}})} \right) \left(\int \mathcal{D}\hat{\psi}_{\text{IR}} \ e^{-i \int \sqrt{-g}\mathcal{L}_{\text{eff}}(\hat{\psi}_{\text{IR}})} \right), \qquad (3.5)$$

that is, the ghost and visible factors identical except for the replacements $\psi \leftrightarrow \hat{\psi}$, $i \to -i$. This demonstrates the matter-renormalization stability of both energy-parity and the decoupling of the ghost and visible sectors.

If we integrate out all massive matter, we arrive at an effective theory of just the photons coupled to the gravitational background,

$$\begin{aligned}
\mathcal{Z}_{\text{eff}} &= \left(\int \mathcal{D}A_\mu \ e^{i \int \sqrt{-g}(-F^2/4-\rho_{\text{vis}})} \right) \left(\int \mathcal{D}\hat{A}_\mu \ e^{-i \int \sqrt{-g}(-\hat{F}^2/4-\rho_{\text{vis}})} \right) \\
&= \left(\int \mathcal{D}A_\mu \ e^{i \int \sqrt{-g}(-F^2/4)} \right) \left(\int \mathcal{D}\hat{A}_\mu \ e^{-i \int \sqrt{-g}(-\hat{F}^2/4)} \right).
\end{aligned} \qquad (3.6)$$

Energy-parity forces the cancellation of the cosmological constant induced by the quantum visible sector, ρ_{vis}, against the corresponding term induced by the ghost sector.

4. Quantum gravity

We next need to consider a cutoff, μ, on graviton momenta, below which we trust eq. (1.1). We will find that in order to adequately suppress gravitational violation of energy-parity as well as vacuum decay, μ must be much smaller than the weak scale energies to which we have tested the SM. Physically, μ represents the scale of unspecified new gravitational physics which serves to cut off amplitudes derived from eq. (1.1). We will further require this physics to be Lorentz-violating. It is unorthodox to contemplate the breakdown of General Relativity at energies below the breakdown of SM quantum field theory, or to consider a fundamental breakdown of Lorentz invariance at any scale, but these ingredients are central to our plot, and we are unaware of any rigorous objections.

Because of energy-parity in the matter sector, the only corrections to the cosmological term are induced by a variety of quantum gravitational corrections. The gravitational sector itself naturally induces a quantum vacuum energy of order μ^4. It is therefore technically natural for the bare cosmological constant in eq. (1.1) to be of this same order,

$$\rho_0 \sim \mathcal{O}(\mu^4). \qquad (4.1)$$

cannot be eliminated. However, as long as there is a conserved fermion number, or fermion number is violated only by $4n$-fermion operators (for example, the Standard Model or the Standard Model with Dirac neutrinos), this extra "i" translates into an overall pre-factor for amplitudes involving odd fermion number processes. Since these do not interfere with amplitudes for even fermion number, the extra "i" falls out of all physical probabilities. Thus, without changing the physics, one can make the replacement $i \to -i$ in front of ghost fermion bilinears relative to visible bilinears.
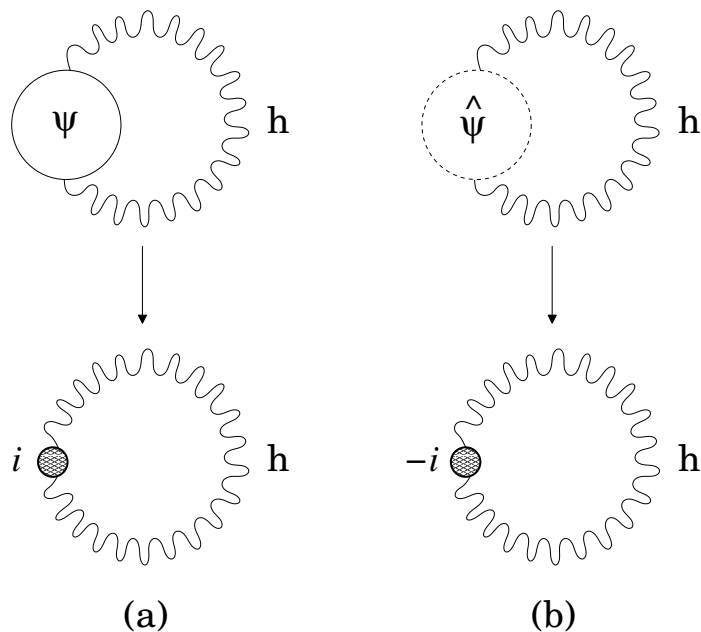
Figure 1: Hard matter (a) and ghost (b) loops attached to graviton loops and their effective vertices.

From now on we will assume this to be the case. Given the observed dark energy $\sim (2 \times 10^{-3} \text{eV})^4$ [18–20], naturalness implies

$$\mu \lesssim 2 \times 10^{-3} \text{eV}. \tag{4.2}$$

This corresponds to a length scale, $1/\mu \sim 100$ microns. A more refined estimate, including factors of (4π), gives a minimal breakdown length of 30 microns [2]. Such a breakdown of General Relativity should be probed by ongoing sub-millimeter tests of Newton's Law which have, so far, probed gravity down to 200 microns [10].

Quantum gravity corrections to matter vacuum energy do not cancel completely, but the largest contributions, from scales above μ, do cancel between the two matter sectors. To see this, note that since graviton momenta are cut off at μ, we can imagine integrating out all matter physics harder than μ before integrating out gravitons. This must generate local effective vertices for the gravity (see figure 1). As discussed in section 3 in path integral terms, the visible and ghost contributions are related by $i \to -i$. For a local vertex, the only factor of "i" is the pre-factor of the action in the path integral. Thus, these hard matter contributions must cancel between the two matter sectors. This leaves integrating out gravitons, cut off by μ, as well as matter physics softer than μ, say from photon loops and ghost-photon loops. These do not generally cancel, because the non-local gravity effective action not only has a pre-factor of i, but also has imaginary parts from soft matter cuts. By dimensional analysis the leading contributions of this type to the cosmological constant are

$$\delta\rho \sim \mu^6/M_{Pl}^2. \tag{4.3}$$

For $\mu \sim \mathcal{O}(10^{-3}\text{eV})$, these contributions are negligible.

There is a caveat in our prediction for the breakdown of Newton's Law in sub-millimeter tests. Inferring the value of the cosmological constant from observations followed by requiring naturalness in the pure quantum gravity sector contribution to the cosmological constant (as above), yields the most accessible length scale for the breakdown of General Relativity. Indeed, we will show elsewhere [17] that these estimates do not change even if one begins with a supersymmetric gravitational sector, given that the SM is not supersymmetric below a TeV. However, it is in principle possible that the gravitational physics that acts to cut off these contributions does not couple appreciably to SM matter, and therefore would not show up in sub-millimeter tests. In this case, there is still a prediction following from the contributions of eq. (4.3), which rests robustly on the coupling of the gravity cutoff physics to matter. Requiring $\delta\rho$ in eq. (4.3) be on the order of the observed dark energy yields a prediction for the breakdown of Newton's Law at distances

$$1/\mu \sim 1/(10\,\text{MeV}) \sim 10 \text{ fm}. \tag{4.4}$$

5. Vacuum decay

We now consider the inevitable instability implied by the full dynamics of the ghost sector coupled to gravity. We assume that in the far past, the ghost sector starts off close to empty, while the visible sector and gravity are close to their state in standard cosmology. The question arises how rapidly physical processes can exploit the negative energy states of the ghost sector in order to populate both that sector as well as the visible and gravity sectors. A similar situation, in the context of "phantom" dark energy [21] was analysed in refs. [6, 7].

To see what issues are involved, let us first return to our toy example of eq. (3.3), now adding a perturbation connecting the regular and ghost-like sectors,

$$\delta\mathcal{H} = g\psi^2\hat{\psi}^2. \tag{5.1}$$

Such a vertex destabilizes the Minkowski vacuum (empty space) by allowing energy conserving processes such as (Nothing) $\to \psi(k_1) + \psi(k_2) + \hat{\psi}(p_1) + \hat{\psi}(p_2)$. At leading order for this process, the event rate per unit time per unit volume is

$$\mathcal{P}_{\to\psi\psi\hat{\psi}\hat{\psi}} \sim g^2 \int d^4p_1 \int d^4p_2 \int d^4k_1 \int d^4k_2 \delta(p_1^2 - m^2)\delta(p_2^2 - m^2)\delta(k_1^2 - m^2)\delta(k_2^2 - m^2)$$
$$\times \; \theta(-p_{10})\theta(-p_{20})\theta(k_{10})\theta(k_{20})\delta^4(p_1 + p_2 + k_1 + k_2). \tag{5.2}$$

This type of calculation resembles ordinary $2 \to 2$ cross-section calculations for $(-p_1) + (-p_2) \to k_1 + k_2$. However, while in $2 \to 2$ scattering the initial momenta, p_1, p_2 are *given* and we only must integrate over the final phase space of k_1, k_2, for vacuum decay we must obviously also integrate over the phase space for p_1, p_2. We will massage this extra phase space integration by defining $P \equiv p_1 + p_2$, $p \equiv p_1 - p_2$, and insert the identity (since P is always time-like or null),

$$\int_0^\infty ds\,\delta(P^2 - s) = 1. \tag{5.3}$$

Further noting that the on-shell δ-functions for the p_i satisfy,

$$\delta((P+p)^2 - m^2)\delta((P-p)^2 - m^2) \propto \delta(p^2 + P^2 - m^2)\delta(P.p), \qquad (5.4)$$

we find

$$
\begin{aligned}
\mathcal{P}_{\to\psi\psi\hat\psi\hat\psi} \sim g^2 \int_0^\infty ds \int d^4 P \delta(P^2 - s)\theta(-P_0) \int d^4 p\,\theta(-p_0)\delta(p^2 + s - m^2)\delta(P.p) \\
\times \int \frac{d^3\vec{k}_1}{2\omega_{k_1}} \int \frac{d^3\vec{k}_2}{2\omega_{k_2}} \delta^4(k_1 + k_2 + P).
\end{aligned}
\qquad (5.5)
$$

Finally, defining $v_\mu \equiv -P_\mu/\sqrt{s}$, we arrive at the simple form,

$$
\begin{aligned}
\mathcal{P}_{\to\psi\psi\hat\psi\hat\psi} \sim g^2 \int \frac{d^3\vec{v}}{2\sqrt{1+\vec{v}^2}} \int_0^\infty ds\,\sqrt{s} \int d^4 p\,\theta(-p_0)\delta(p^2 + s - m^2)\delta(-p.v) \\
\times \int \frac{d^3\vec{k}_1}{2\omega_{k_1}} \int \frac{d^3\vec{k}_2}{2\omega_{k_2}} \delta^4(k_1 + k_2 - \sqrt{s}v),
\end{aligned}
\qquad (5.6)
$$

where v_μ has become a 4-velocity with $v_0 \equiv \sqrt{1+\vec{v}^2}$.

Notice that for *fixed* v and s, the remaining phase space integrals over $p, \vec{k}_1$ and $\vec{k}_2$ are necessarily finite. In particular they give some Lorentz-invariant function of s and v_μ — really a function of s alone since $v^2 = 1$. For $s \gg m^2$, this function scales as $\sqrt{s}$ by dimensional analysis. Thus, the integral over s has a serious power divergence even in this tree level calculation. Assume that some new Lorentz-invariant physics appears at a scale $s_{\max}$ to cut off this divergence and give a finite s integral. This still leaves the integral over $\vec{v}$, which diverges quadratically because of the v-independence deduced above. There is no way for any new Lorentz-invariant physics to cut off this divergence in the decay probability. To get a finite answer we must have Lorentz-violating physics act as the cutoff [7] at a scale $\mathcal{E}$, and assume Lorentz-invariance is a (very good) approximate symmetry below this scale. Thus, in our toy example, we estimate

$$\mathcal{P}_{\to\psi\psi\hat\psi\hat\psi} \sim g^2 \mathcal{E}^2 s_{\max}. \qquad (5.7)$$

The decomposition of the phase space integrals for the decays of the vacuum seen in the above example generalize straightforwardly to all varieties of such processes at any order in perturbation theory. There is an overall integral over the total ghost 4-momentum,

$$\int d^4 P \ldots = \int d^3\vec{v}/2\sqrt{1+\vec{v}^2} \int ds\, s \ldots, \qquad (5.8)$$

with all phase space integrals over relative ghost momenta and all visible momenta yielding some finite function of s alone if all physics is Lorentz-invariant. New Lorentz-invariant physics can cut off the s integral, but there is always an overall divergent $\int d^3\vec{v}/2\sqrt{1+\vec{v}^2}$ which can only be cut off by invoking high-energy Lorentz-violation. Since, in our scenario, such processes connecting the ghost and other sectors always go via the gravitational sector, we will identify both $\sqrt{s_{\max}}$ and $\mathcal{E}$ with μ.[3]

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[3]In principle, we could simply consider two gravitational cutoffs, $\sqrt{s_{\max}} \neq \mathcal{E}$, which would change some of our estimates. We have adopted a single gravitational cutoff scale, μ, as the simplest option in this paper. We discuss the implications of the alternative option in future work [17].

The dominant two processes for vacuum decay are

$$\text{Nothing} \rightarrow \psi + \psi + \hat{\psi} + \hat{\psi},$$
$$\text{Nothing} \rightarrow (\text{graviton} - \text{excitation}) + \hat{\psi} + \hat{\psi}. \tag{5.9}$$

The first process is mediated by an off-shell graviton, so its decay rate is suppressed by $1/M_{Pl}^4$. The second process includes the possibility of producing excited gravitons (or other particles in the gravitational sector) responsible for cutting off gravity at the scale μ. The first process is the dominant production of visible matter from the vacuum. For the case of massless ψ (i.e., photons), we use dimensional analysis, and include the (4π)s which result from phase space integrals to estimate

$$\mathcal{P}_{\rightarrow \gamma\gamma\hat{\gamma}\hat{\gamma}} \sim \frac{1}{4\pi} \left(\frac{1}{8\pi^2} \right)^2 \frac{\mu^8}{M_{Pl}^4}$$
$$\sim 2 \times 10^{-92} \left(\frac{\mu}{2 \times 10^{-3}\text{eV}} \right)^8 [\text{cm}^3 \times 10\text{Gyr}]^{-1}. \tag{5.10}$$

Given our estimate of μ, the number of photons produced over the lifetime of the universe is completely negligible. We become experimentally sensitive to this process in the cosmic ray background [22] only when $\mu \gtrsim \text{MeV}$ [7]. This may become relevant if the possibility of a higher gravitational cutoff, as discussed at the end of the last section, is realized.

The second process, the decay into excited gravitons and ghosts, is the dominant process populating the universe with ghosts. We take the gravity-sector particles to have mass $\sim \mu$ and their coupling to (ghost) matter of gravitational strength. A rough estimate of the current energy density of ghost radiation due to this decay is the rate times the age of the universe times the average energy of the ghosts (which again will be of order μ). The estimate

$$t_0 \, \mu \, \mathcal{P}_{\rightarrow h^*\hat{\gamma}\hat{\gamma}} \sim \frac{1}{4\pi} \left(\frac{1}{8\pi^2} \right) \frac{\mu^7}{M_{Pl}^2} t_0$$
$$\sim (2 \times 10^{-12}\text{eV})^4 \left(\frac{\mu}{2 \times 10^{-3}\text{eV}} \right)^7 \left(\frac{t_0}{10\text{Gyr}} \right), \tag{5.11}$$

is negligible compared with, for example, the radiation energy density today.

We have focused on photons as massive particles are even less important due to the inherent phase-space cutoff μ on all processes.

6. Classical gravity with ghosts

In a fixed external gravitational field such as we considered in section 3, ghost matter behaves identically to ordinary matter, the only distinction being the overall sign of the Lagrangian, which is irrelevant to the equations of motion. For example, if a ghost particle is brought near the Earth it will fall towards the ground. A ghost mass and a visible mass will be repelled from each other, however, if the ghost mass dominates. This is because we can think of the ghost mass as setting up a gravitational field which is then felt by the smaller visible mass. Since the sign of the ghost stress tensor is reversed, the linearized gravitational field set up is also reversed. The visible mass sees this reversed field, and

is repelled rather than attracted. Thus, there must be a transition from attraction to repulsion depending on the masses. The gravitational force between two ghosts is also repulsive. To see this, note that since the overall sign of the action is irrelevant, it is simpler to think of the gravitational sector as being ghost-like relative to the ghost sector. The non-relativistic force law arises diagrammatically from one-graviton exchange. The relative ghost-like nature of gravity implies a sign-flip for the graviton compared to the usual computation. In the ghost sector, therefore, the usual law of universal attraction is replaced by universal repulsion.

All these results are illustrated by writing the Lagrangian in the non-relativistic approximation for the relative motion between two masses:

$$L_{\text{relative}} = \frac{1}{2} \frac{m_1\, m_2}{m_1 + m_2} \dot{\vec{r}}^2 + G_N \frac{m_1\, m_2}{|\vec{r}|}. \tag{6.1}$$

The kinetic coefficient is just the usual formula for the reduced mass. This formula continues to hold even in the presence of ghost masses, the only difference being that these ghost masses must be considered *negative*. There are, of course, relativistic corrections to the static force, most significantly gravitational radiation from accelerating ghost masses. Instead of slowing such masses, the emission of gravitational radiation speeds them up.

The center-of-mass coordinate is cyclic and decouples from the relative motion as usual. However, in the limit $m_1 = -m_2 \equiv m$, the relative coordinate $\vec{r}$ becomes cyclic and the center of mass coordinate becomes proportional to $\vec{r}$. The more useful coordinate is the average position $\vec{R} = \vec{x}_1 + \vec{x}_2$ with the equation of motion

$$\ddot{\vec{R}} = \frac{2G_N m}{r^2} \hat{r}, \tag{6.2}$$

in which case the matter-ghost system spontaneously accelerates in the direction $\hat{r}$, while their relative positions remain fixed.

To avoid such an exotic type of dark matter (which would spoil standard cosmology were it to (co-)dominate), we require the ghost sector to be far more empty than the visible sector of our universe. This should be considered a (plausible) requirement on the initial conditions of the universe. As long as the negative energy density and pressure of ghost matter is subdominant, the expansion of the universe is driven by visible matter, and the cosmological term in standard fashion.

There is a constraint on how far back in cosmological time our effective theory continues to make sense, following from the gravitational cutoff μ^2 on spacetime curvature. From Einstein's Equations this corresponds to an energy density in the matter sector of order $\mu^2 M_{Pl}^2 \sim \text{TeV}^4$. Thus we are constrained to cosmology from roughly just above the electroweak phase transition to the present.

7. Metastable matter vacua

In any proposal which claims to attack the cosmological constant problem, one must consider what happens when the matter sector has a metastable vacuum as well as a true vacuum, in order to understand what principle determines which vacuum has the sup-
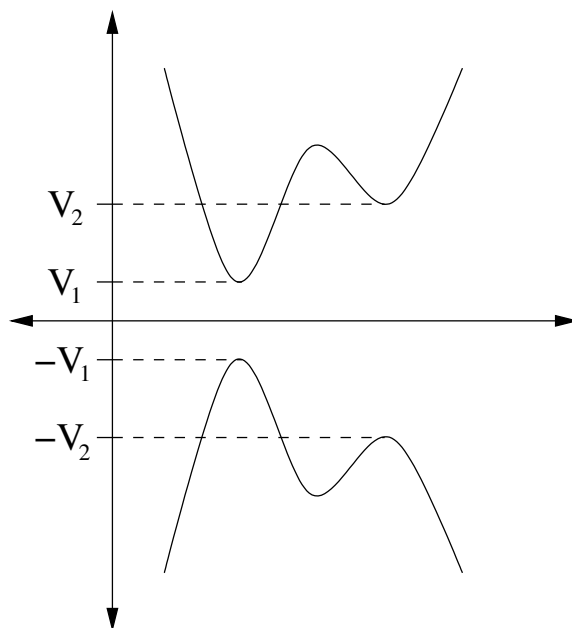
Figure 2: Meta-stable vacua in a theory with energy parity.

pressed cosmological constant. In the present scenario, a metastable vacuum in the visible sector must be reflected in the ghost sector, as illustrated in figure 2. The corresponding vacuum energies are taken to be $V_2 > V_1$ and $-V_2 < -V_1$. When the two sectors occupy parity-symmetric vacua at $\pm V_1$ or $\pm V_2$, the matter contribution to the cosmological constant vanishes (up to the small quantum gravitational corrections we estimated earlier). But if the visible sector is at V_2 while the ghost sector is at $-V_1$, the matter contribution to the cosmological constant is $V_2 - V_1 > 0$. Similarly, if the visible sector is at V_1 while the ghost sector is at $-V_2$, a negative cosmological constant, $V_1 - V_2$ emerges for the lifetime of the metastable state. If the metastable vacua decay in the long run (before the vacuum energy becomes dominant), then eventually both matter sectors will be near $\pm V_1$ where the matter contribution to the cosmological constant vanishes.

8. Inflation

The desired initial conditions of our scenario — standard big bang cosmology beginning before nucleosynthesis and an empty ghost sector — is surprisingly easy to achieve using cosmic inflation. Here we elaborate on ref. [3]. The continuity equation for the ghost sector is the same as that for ordinary matter. For an approximately homogeneous and isotropic universe, the equation is

$$0 = T^{\mu}_{\nu;\mu} \simeq \frac{\partial \rho_{\text{ghost}}}{\partial t} + 3(\rho_{\text{ghost}} + p_{\text{ghost}})\frac{\dot{a}}{a}, \qquad (8.1)$$

where ρ_{ghost} and p_{ghost} are the energy density and pressure of the ghost sector and a is the scale factor in the Robertson-Walker metric. While ρ_{ghost} (and for radiation, p_{ghost}) is negative, the energy density clearly scales like that of normal matter and radiation, namely
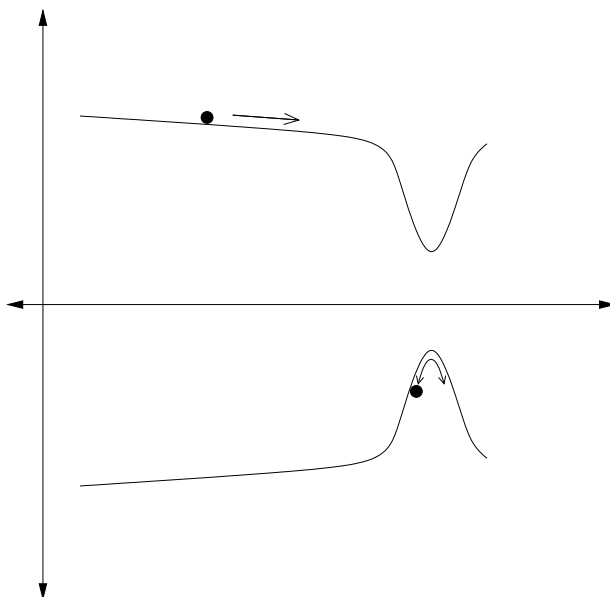
Figure 3: Inflation in a theory with energy parity.

$\rho_{\mathrm{ghost}} \sim a^{-4}$ for radiation and $\rho_{\mathrm{ghost}} \sim a^{-3}$ for non-relativistic matter. Thus, if the universe were in a state in which positive vacuum energy dominated, both normal radiation and matter and ghost radiation and matter would dissipate due to the exponentially growing scale factor.

Inflation can be generated by the displacement of a scalar field from its minimum as long as its potential is flat enough (see figure 3). One necessity is that the corresponding field in the ghost sector is sitting at its maximum for the number of e-foldings required by inflation. In fact, the ghost inflaton could be initially displaced, so long as its displacement is smaller than that of the inflaton and positive vacuum energy dominates. The ghost partner's dynamics will be governed by eq. (8.1) with $\rho_{\mathrm{ghost}} = -(\partial_t \hat{\phi})^2 - (\nabla \hat{\phi})^2/(2a^2) - V(\hat{\phi})$ and $p_{\mathrm{ghost}} = -(\partial_t \phi)^2 + (\nabla \phi)^2/(2a^2) + V(\hat{\phi})$, leading to

$$\ddot{\hat{\phi}} + 3\frac{\dot{a}}{a}\dot{\hat{\phi}} + V'(\hat{\phi}) = 0, \tag{8.2}$$

where $V(\hat{\phi})$ is the potential function in $\mathcal{L}_{\mathrm{matt}}$, and is thus the same in the two sectors. The ghost inflaton has the same equation of motion as the visible sector inflaton, but because of the overall minus sign, the actual potential for the ghost inflaton is inverted compared with that of the visible sector. The displaced ghost rolls *up* the potential towards its maximum, performs coherent oscillations and/or reheats into ghost radiation. Positive vacuum energy dominates and the ghost radiation dilutes as the universe inflates. Finally, the standard inflaton rolls towards its minimum, perhaps performs coherent oscillations, and then reheats an otherwise empty universe with only visible matter. This symmetric looking evolution of the ghost inflaton is what one expects — dynamical gravity breaks the energy parity symmetry, but coupling to the background metric does not.

In the simplest picture, the model-building challenge is only to construct an inflation scenario in which the curvature (and therefore the Hubble scale) during inflation is less than μ, the energy-momentum cutoff of gravity. Speculation about, for example, the physics responsible for the cutoff μ may lead to possibilities outside of inflation which address the question of initial conditions of our universe.

9. Discussion

We proposed a simple symmetry to control the cosmological constant, but one with an apparently fatal flaw, the instability of flat spacetime. The danger from this instability, however, is entirely sensitive to features of short distance gravity outside the currently probed experimental regime. This scenario, therefore, changes the character of the cosmological constant problem. The strictest bounds on the distances at which gravity must be modified in fact arise from the explicit breaking of the protective symmetry by gravity, putting such modifications within reach of ongoing testsof short distance gravity [10 – 13]. Controlling the instability does, however, introduce a new qualitative requirement, namely gravitational Lorentz-violation. This is (model-dependently) another source of potential experimental signals [8]. Gravitational Lorentz violation can also radiatively induce Lorentz violation in visible matter, but these effects (in fractional shifts in maximal speeds) are negligibly small, $\lesssim \mathcal{O}(\mu^2/M_{Pl}^2) \sim 10^{-60}$. Our scenario has an acceptable cosmology provided we have initial conditions with the ghost sector very sparsely populated, and we showed how inflation can make these initial conditions natural. Ghost matter has unusual gravitational laws and unusual equations of state. If enough of it has survived to the present, it may provide interesting signals in precision cosmological measurements.

It is of course important to understand how to build consistent theories with gravitational breakdown energy scales far below that of non-gravitational particle physics, as well as how to incorporate ghost matter at the fundamental level. For gravitational Lorentz violation, there are two possible scenarios. First, Lorentz invariance may not be a fundamental symmetry of Nature, but rather some sort of accidental or emergent symmetry. Of course this implies emergent General Relativity [23] of some sort. Second, it may be that Lorentz invariance is a fundamental symmetry, but the gravitational vacuum spontaneously breaks this symmetry. Gravitational fluctuations about such a vacuum need not be constrained by exact Lorentz invariance, as exemplified by the effective field theories in [8]. Exploration of the character of this Lorentz-violating cutoff may hold the key to additional experimental tests of our scenario.

Acknowledgments

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